Int. J. Multiphase Flow. Vol. 4, pp. 81–88. Pergamon/Elsevier. 1978. Printed in Great Britain.

SHOCK-WAVE FORMATION IN FLOWING BUBBLY MIXTURES BY STEEPENING OF COMPRESSION WAVES

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(*Received* 1 *February* 1977)

Abstract—The formation and propagation of shock waves in a two-component flowing bubbly mixture has been investigated experimentally. The structure of shock waves formed by steepening of compression waves is compared with the corresponding features of shocks produced spontaneously in shock tubes. Experimentally determined values of the speed of propagation of the shock compare favorably with the Hugoniot relationship based upon a homogeneous two-phase model. The effect of the gravitational and frictional pressure gradients on the shock characteristics is also examined.

1. INTRODUCTION

It is well known that small amounts of free gas in a liquid can cause the acoustic velocity in the medium to be reduced considerably from that in the pure liquid itself. For a mixture of gas bubbles and liquid the effect of the gas phase on the acoustic velocity has been well documented in the literature by numerous investigators, and in particular by Hsieh & Plesset (1961), Henry (1969), and van Wijngaarden (1968). Inasmuch as the effect of compressibility is governed by the gas phase, the acoustic velocity in a bubbly mixture is influenced by both the pressure and void fraction α. For certain void fractions it can be shown (Henry 1969) that the acoustic velocity based on a homogeneous model can be approximated by

$$a = \left[\frac{np}{\rho_L\alpha(1-\alpha)}\right]^{1/2} \qquad [1]$$

where p is the pressure, ρ_L the density of the liquid, and n is the polytropic exponent.

The propagation speed of small-amplitude pressure pulses is equal to the acoustic velocity, and hence can be represented by [1]. As the propagation speed increases with an increase in pressure and a decrease in void fraction, a compression wave propagating in a gas–liquid mixture undergoes a continuous change in form due to the steepening of its front, a process which may ultimately result in the formation of a shock wave. In contrast to waves generated in shock tubes, shocks formed in flowing bubbly mixtures may undergo even further changes in shape as a result of prevailing gravitational and frictional pressure gradients.

Shock-wave propagation in gas–liquid mixtures in shock tubes has been studied by Campbell & Pitcher (1958) and Noordzij & van Wijngaarden (1974). Campbell & Pitcher (1958) established that the propagation speed of shock waves is higher than the acoustic speed and is dependent on the strength of the shock wave p_1/p_0, where p_0 and p_1 denote the pressure in front of and to the rear of the shock wave, respectively. For isothermal bubble behavior, they derived the Hugoniot relationship for a stationary shock wave in a gas–liquid mixture to be

$$M_0^2 = p_1/p_0 \qquad [2]$$

where $M_0 = U/a_0$, U is the shock speed, and a_0 is the homogeneous acoustic velocity in the mixture at pressure p_0. For adiabatic behavior, van Wijngaarden (1972) reports the corresponding relationship

$$M_0^2 = \frac{1}{\gamma}\frac{(p_1/p_0) - 1}{1 - (p_0/p_1)^{1/\gamma}} \qquad [3]$$

where γ is the ratio of specific heats. The size and distribution of gas bubbles in a bubbly mixture can have an appreciable influence on the wave propagation, as reported by Plesset (1964), who concluded that oscillations behind the shock are governed by isothermal behavior at both low and high frequencies, but by adiabatic behavior over an intermediate range.

The structure of a shock wave in a bubbly gas–liquid mixture is characterized by a steep-rising frontal region, pressure oscillations at the rear of the shock, and a smooth region over which the pressure gradually attains its final value. Noordzij & van Wijngaarden (1974) classified shocks as types A, B, and C according to the phenomena of compression, frequency dispersion, and dissipation, all of which are associated with the radial and translational relative motion of the bubbles. In gas dynamics it is well known that a compression wave steepens due to convection, ultimately resulting in a steady shock wave if wave steepening is balanced by viscous dissipation. In the case of bubbly mixtures, however, the steepening, also referred to as amplitude dispersion, is mainly caused by wave compression. This steepening process is resisted by two other phenomena, frequency dispersion and dissipation. The frequency dispersion is due to the existence of a pressure difference across the gas bubble, and the associated radial oscillations. The relative translational and radial motions of bubbles are resisted by the viscous dissipation effect of the liquid. Dissipation mechanisms due to thermal conduction and acoustic radiation may also exist.

Most if not all previous investigations on shock-wave formation in mixtures have been conducted in shock tubes, for which there is no mean mass flow of the liquid component. The subject of this paper is the formation of shock waves in a bubbly mixture by steepening of compression waves. The effect of the flowing mixture on shock structure will also be discussed.

2. EXPERIMENTAL APPARATUS

The experimental setup consisted of an 18.75 m long, 2.60 cm dia. clear plastic pipe with a wall thickness of 0.63 cm. The piping is in the shape of an inverted U, as shown in figure 1. Water enters the pressurized reservoir and then flows vertically upward through the streamlined entrance, which is always submerged. Compressed air is injected into the flowing water through a porous wall made of sintered stainless steel. The mixture then flows vertically upward in the test pipe to the first long-radius elbow, which turns the flow into the horizontal leg, which in turn is connected to the vertical downcomer. The radius of curvature and the total length of each long-radius elbow, which were made of PVC, were 0.279 m and 0.457 m, respectively. The rate of water flow is controlled by the exit gate valve as well as by an inlet valve. The transient reported in this paper was generated by a spring-operated quick-acting gate valve at the exit. By proper adjustment of the spring the duration of valve closure could be varied from 5 to 100 msec.

The volumetric flow rate of the water was measured by a bend meter. A micrometering valve with a fine adjustment was used to meter the mass flow rate of the compressed air. Pressure transducers could be flush-mounted at any of the thick flanges, which were placed every 1.22 m along the pipe. The pressure sensors were Kulite Model XTM-1-190 semiconductor transducers with a 0.25 cm diameter sensing area. The pressure range and natural frequency were 1.7 MPa and 125 kHz, respectively.

3. FLOW REGIMES AND BUBBLE-SIZE DISTRIBUTION

Preliminary tests were conducted for the identification of the various two-phase flow regimes and for the determination of bubble size distribution. As seen in the photograph of

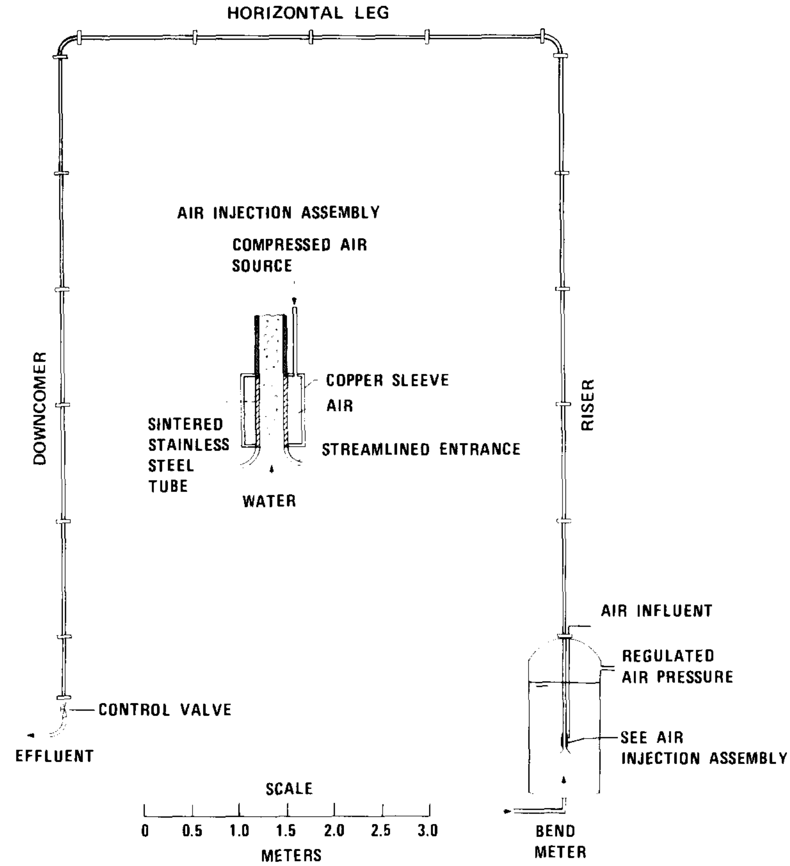

Figure 1. Experimental apparatus.

figure 2, the riser has a more or less uniform distribution of bubbles across the entire cross section. The bubbles are well rounded, but not of uniform size, ranging in diameter from 0.5 to 2 mm. As the initial steady-flow pressure distribution did not vary much for the experiments reported herein, the main influence on the bubble size was the void fraction. It was observed that as the void fraction was increased the average bubble-size also increased, but only slightly. The average bubble-size in the riser varied from 1 to 1.5 mm for the range of $0.005 < \beta < 0.08$, in which β is the volumetric quality.

In the horizontal portion, as shown in figure 2, the bubbles occupy the top portion of the pipe and to some extent coalescence takes place. Also, it may be noted that the bubbles lose their spherical shape while becoming larger in size.

In the vertical downcomer the bubbles are large and mainly concentrated along the center of the pipe. They appear to flow intermittently with a tendency to form clusters. Even though the bubbles are large and non-spherical they are more or less of uniform size, with an average dimension of about 3 mm.

4. RESULTS

4.1 *Shock-wave formation*

The photographs of oscilloscope pressure traces shown in figure 3 illustrate the phenomenon of steepening of compression waves created by the rapid closure of the exit valve in the pipe. The pressure transducers were located very close to the valve (P_1) and 1.98 m upstream of it (P_2). For the entire range of volumetric qualities shown in figure 3 steepening of the wave front is apparent. A weak shock is formed in figure 3a as the strength of the wave, p_1/p_0, is relatively small. The high degree of steepening noticeable in figure 3b exhibits the significant features of a

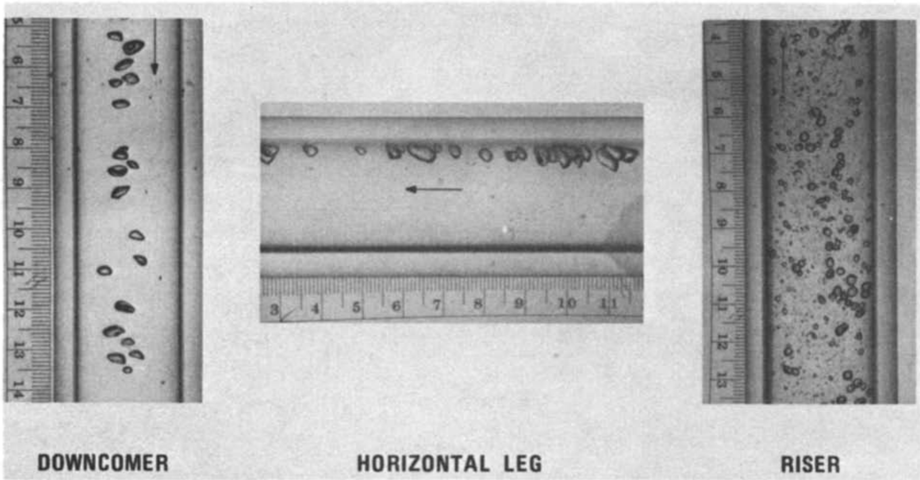

Figure 2. Typical bubble distribution in three legs of test pipe. Smallest division on each scale is 1 mm.

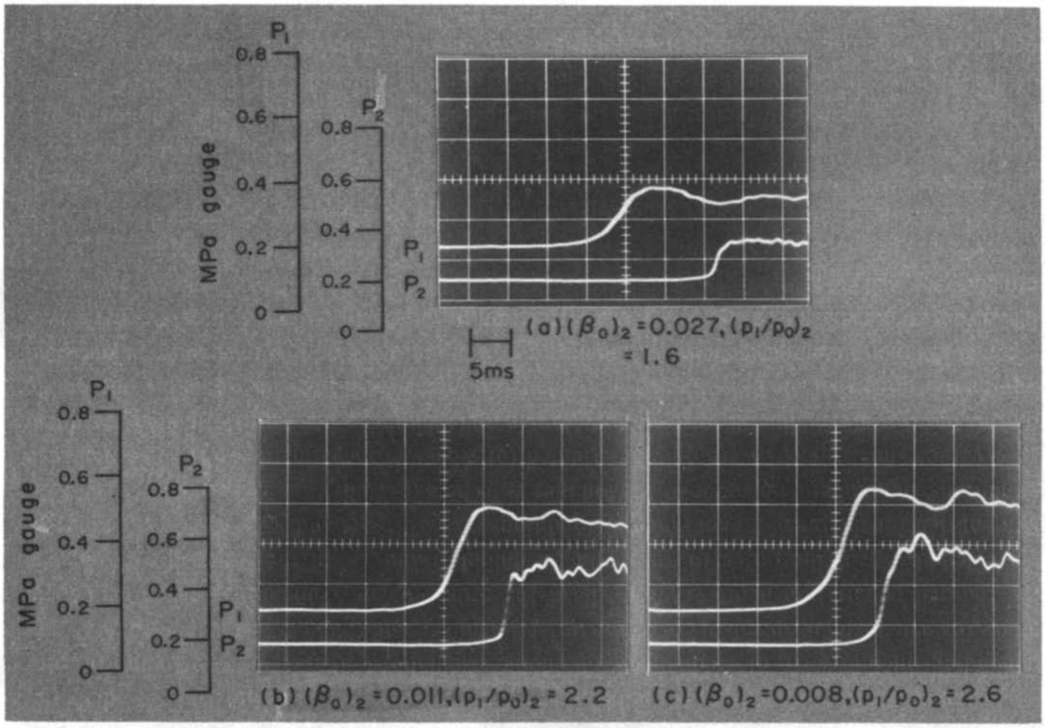

Figure 3. Steepening of compression waves. Piezometers P_1 and P_2 at $x = 18.45$ m and $x = 16.77$ m, respectively. Mean liquid velocity $u_0 = 1.34$ m/sec.

shock with a steep front and damped oscillations. The lower quality or void fraction for the test shown in figure 3c resulted in a shock of greater strength and speed of propagation. Because of the greater speed the compression wave has not yet fully steepened in its short travel of 1.68 m. Nevertheless, the steepening phenomenon is very significant.

The steepening process is dependent on the pressure, the void fraction, and the form of the initial compression wave, which in this study is governed by the time of closure of the valve. Obviously, both the distance over which the steepening takes place and the consequent formation of shock waves are affected by the above factors, which also influence shock characteristics. The time of closure of the valve has a significant influence on the shock formation as a compression wave of greater thickness takes more time to transform into a shock wave.

4.2 *Speed of propagation of shock waves*

In the case of bubbly mixtures the behavior of the bubbles may be isothermal or adiabatic, depending on the range of frequencies of bubble oscillations, Plesset (1964). In the present study for which shock speeds were of the order 100 m/sec and thicknesses of the order 10^{-2} m, the thermal penetration depth $(D_G/\omega)^{1/2}$ is of the order 10^{-4} m, compared with an acoustical wave length λ_G of the order 10^{-2} m. With the bubble radius R of the order of 10^{-3} m, it is evident that for the present experimental conditions $(D_G/\omega)^{1/2} \ll R$ and $\lambda_G \gg R$. For this range of conditions an adiabatic behavior of the bubbles is to be expected.

Average shock speeds were only determined for waves propagating down the riser because of the more uniform bubble distribution therein than in the other legs. Although the local volumetric quality of air β_0 is known at the corresponding local pressure p_0, the value of the local void fraction α_0 is needed for substitution of a_0 into [2] or [3]. Inasmuch as the void fraction was not measured the following correlation proposed by Zuber & Findlay (1965) was utilized

$$\frac{\beta}{\alpha} = C_0 + \frac{V_{Gj}}{j} \qquad [4]$$

where C_0 is a distribution factor, V_{Gj} is the weighted drift velocity and j is the volumetric flux of the mixture. Based upon their analysis C_0 was assigned a value of 1.2. The value of V_{Gj} was obtained for the riser from another empirical relationship of Zuber & Findlay (1965)

$$V_{Gj} = 1.38\left[\frac{\sigma g \Delta\rho}{\rho_L}\right]^{1/4} \qquad [5]$$

where σ is the surface tension of the liquid, g is the gravitational acceleration, and $\Delta\rho = \rho_L - \rho_G$. Using [1], [2] and [3] theoretical shock speeds were calculated for average values of p_1/p_0 obtained from the photographs. The experimental shock speeds are compared with adiabatic and isothermal theory by using an average value of p_0 in [1] with the appropriate value of n. The results shown in figure 4 suggest that adiabatic gas behavior yields a slightly better correlation with experiment. In the computation of the wave speed results depicted in figure 4 the effect of gravity over the short distance (1.22 m) over which the shock speed was measured was neglected.

4.3 *Structure of shock waves*

For volumetric qualities ranging from $0.005 < \beta < 0.05$, for initial steady-flow water velocities $1.2 < U_0 < 1.7$ m/sec, shock waves of weak to moderate strength, $1.4 < p_1/p_0 < 3.2$, were observed. The photographs shown in figures 5 and 6 represent typical shocks at different locations along the pipe. Each photograph illustrates the shock profile as the wave propagates up the downcomer (figure 5), or down the riser (figure 6). In all cases the pressure recordings P_1 and P_2 are 2.44 m apart.

The photographs of figures 5 and 6, as well as many others that are not reported herein, suggest that type A shocks, which are characterized by a very steep front with the peak pressure above the final equilibrium value, did not occur in the present study. This is in contrast to the shock-tube experiments of Noordzij (1973), for which a shock was generated almost instantaneously, resulting in a very steep front which continuously changed along the tube. In the present study, however, a compression wave of moderate thickness transforms itself in a shock wave by steepening. The steepening process is opposed by the viscous resistance to the

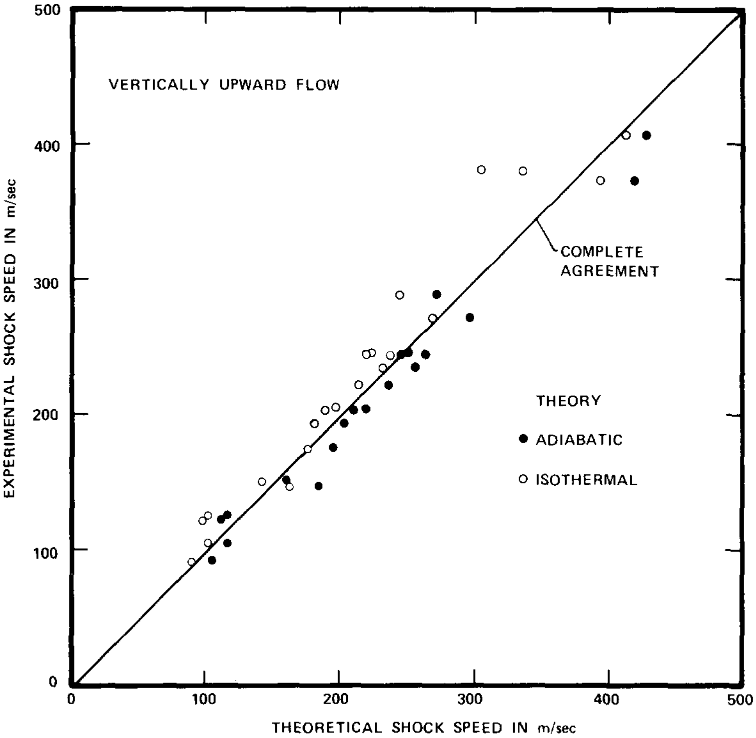

Figure 4. Comparison of experimental and theoretical shock speeds in riser.

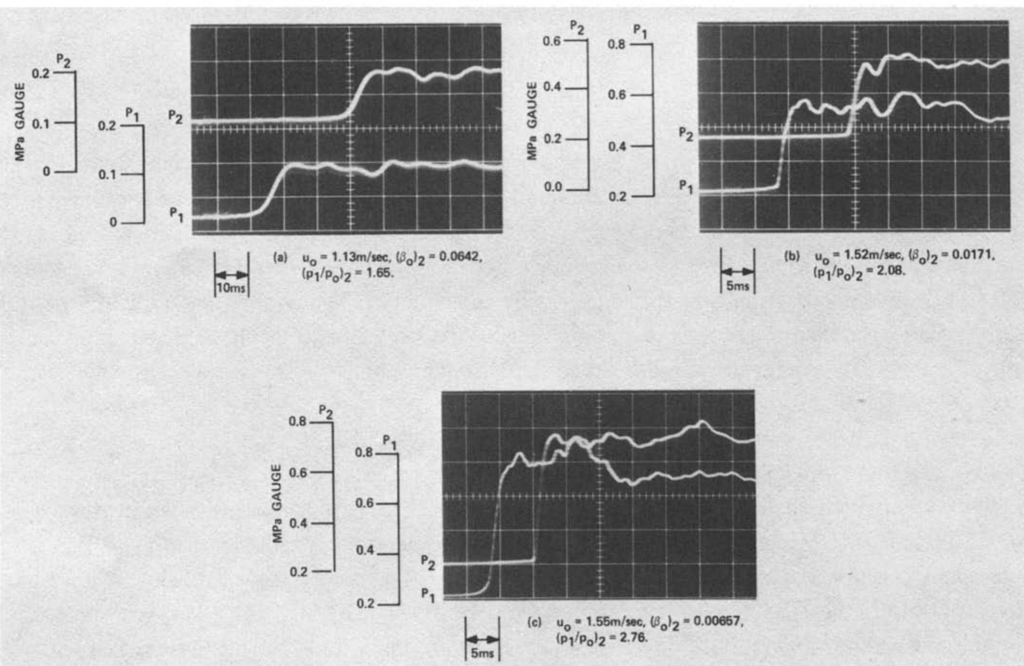

Figure 5. Profiles of shock waves propagating up the downcomer. pressure traces P_1 and P_2 measured at $x = 15.55$ m and $x = 13.11$ m, respectively.

radial and translational relative motions of the bubbles, as well as by dispersive effects. Hence, the physical process of shock formation by steepening of a compression wave makes the shock waves of this study different from those generated in shock tubes. For the present study the typical time of closure for spring-activated closure of the valve of 10 ms was apparently long enough to preclude the formation of type A shocks.

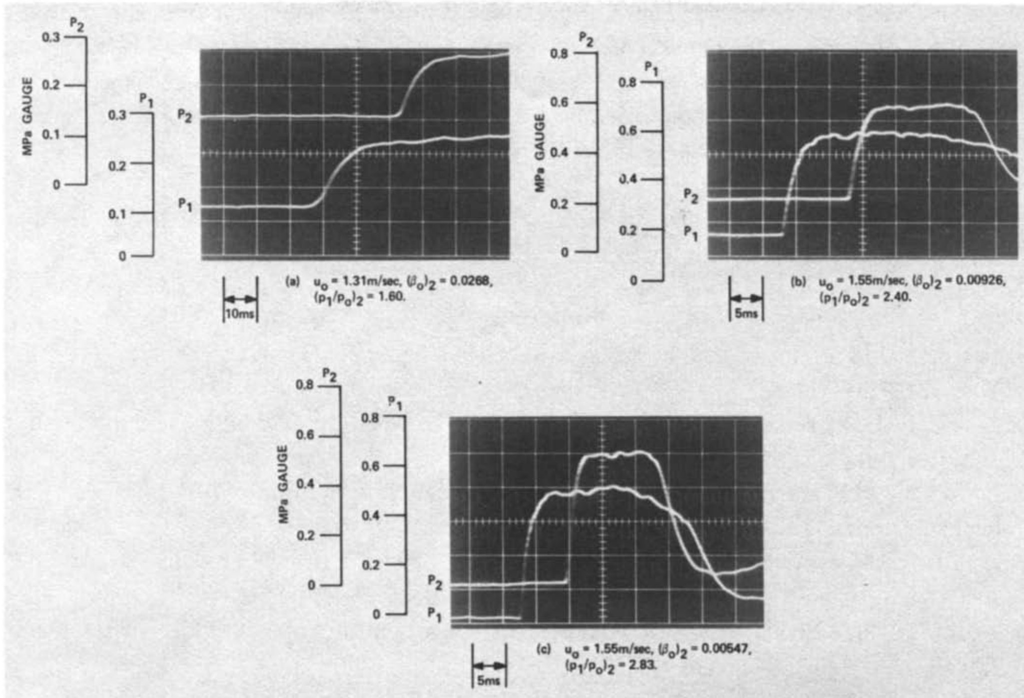

Figure 6. Profiles of shock waves propagating down the riser. Pressure traces P_1 and P_2 measured at x = 4.88 m and x = 2.44 m, respectively.

Figure 5a shows a weak shock propagating from P_1 3.20 nm from the valve to a location P_2 5.64 m from the valve. It is noted that there is very little steepening of the wave front over this 2.44 m of travel, and that the form of the shock appears practically unchanged. The weak shock propagating down the riser in figure 6a is similar to type C shocks observed by Noordzij (1973) as its front is gradually rising and as there are practically no oscillations behind the shock. Steepening of the front is seen to persist even after the wave has traveled almost 16.31 m from the valve. Figures 5a and 6a illustrate that, for higher void fractions, the steepening phenomenon is less pronounced as the change in wave celerity is relatively low.

Figure 5b shows a shock of moderate strength, $p_1/p_0 = 2.08$. The shock front is initially steep, followed by irregular oscillations, resembling the type B shocks observed by Noodzij. The shock wave of figure 6b of moderate strength, $p_1/p_0 = 2.40$, also appears to be in equilibrium as it propagates down the riser. The oscillations behind this shock are extremely small compared to those of figure 5b, suggesting an influence of bubble size as well as shape. The irregularity in the oscillations were not unexpected because of the nonuniformity in bubble-size distribution. As the bubbles are smaller and more numerous in the riser than in the downcomer, the amplitudes of the oscillations are very small, and consequently damp out quite rapidly.

A shock wave of strength $p_1/p_0 \simeq 2.8$ is shown propagating up the downcomer in figure 5c, and then down the riser in figure 6c. Figure 5c shows that, for lower void fractions the steepening phenomenon is initially very pronounced in the downcomer. As the shock speed is higher the steepening is observed over a considerable length of the pipe. The shock does assume a more or less equilibrium profile as it travels upstream. It should be noted that, even for a strength of $p_1/p_0 \simeq 2.8$, the oscillations behind the shock wave are very weak in the riser.

For shock-wave photographs obtained over a range of air concentrations for which the strength p_1/p_0 ranged from 1.4 to 3.2, no significant changes in the shock structure were observed, except that the oscillations behind the shock appeared to be strong in the downcomer and very weak in the riser. Noordzij (1973) observed significant changes in the structure of a shock wave as it propagated down a shock tube containing a bubbly mixture. However, in most of his experiments the liquids used had a much higher viscosity than that of water.

The effect of the frictional and the gravitational pressure gradients on shock structure appears to be negligible when comparing the pressure traces of equilibrium shocks; e.g. figures 5b and 5c. Similar conclusions were stated by Noordzij (1973) for shock-tube experiments.

Acknowledgement—The investigation presented in this paper was supported in full by the National Science Foundation through Grant GK-38570, Georgia Institute of Technology.

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